

The effect of primordial non-Gaussianity on the topology of large-scale structure

C. Hikage,^{1,2*} P. Coles,² M. Grossi,³ L. Moscardini,^{4,5} K. Dolag,³ E. Branchini⁶ and S. Matarrese^{7,8}

¹*School of Physics and Astronomy, University of Nottingham, University Park, Nottingham NG7 2RD*

²*School of Physics and Astronomy, Cardiff University, 5, The Parade, Cardiff CF24 3AA*

³*Max-Planck-Institut für Astrophysik, Karl-Schwarzschild Strasse 1, D-85748 Garching, Germany*

⁴*Dipartimento di Astronomia, Università di Bologna, via Ranzani 1, I-40127 Bologna, Italy*

⁵*INFN, Sezione di Bologna, viale Bertini 6/2, I-40127 Bologna, Italy*

⁶*Dipartimento di Fisica, Università di Roma TRE, via della Vasca Navale 84, I-40127 Bologna, Italy*

⁷*Dipartimento di Fisica, Università degli Studi di Padova, via Marzolo 8, I-35131 Padova, Italy*

⁸*INFN, Sezione di Padova, via Marzolo 8, I-35131 Padova, Italy*

Accepted 2008 January 9. Received 2008 January 8; in original form 2007 November 21

ABSTRACT

We study the effect of primordial non-Gaussianity on the development of large-scale cosmic structure using high-resolution N -body simulations. In particular, we focus on the topological properties of the ‘cosmic web’, quantitatively characterized by the Minkowski functionals (MFs), for models with quadratic non-linearities with different values of the usual non-Gaussianity parameter f_{NL} . In the weakly non-linear regime (the amplitude of mass density fluctuations $\sigma_0 < 0.1$), we find that analytic formulae derived from perturbation theory agree with the numerical results within a few per cent of the amplitude of each MF when $|f_{\text{NL}}| < 1000$. In the non-linear regime, the detailed behaviour of the MFs as functions of threshold density deviates more strongly from the analytical curves, while the overall amplitude of the primordial non-Gaussian effect remains comparable to the perturbative prediction. When smaller-scale information is included, the influence of primordial non-Gaussianity becomes increasingly significant statistically due to decreasing sample variance. We find that the effect of the primordial non-Gaussianity with $|f_{\text{NL}}| = 50$ is comparable to the sample variance of mass density fields with a volume of $0.125(h^{-1} \text{ Gpc})^3$ when they are smoothed by Gaussian filter at a scale of $5 h^{-1} \text{ Mpc}$. The detectability of this effect in actual galaxy surveys will strongly depend on residual uncertainties in cosmological parameters and galaxy biasing.

Key words: methods: analytical – methods: N -body simulations – methods: statistical – early Universe – large-scale structure of Universe.

1 INTRODUCTION

According to the standard scenarios for the formation of large-scale structure in the Universe, the present-day cosmic density field evolves from small-amplitude initial fluctuations which are described by Gaussian statistics. The hypothesis of primordial Gaussianity is supported by present observations of the cosmic microwave background (CMB), particularly those from the *Wilkinson Microwave Anisotropy Probe* (WMAP) (Komatsu et al. 2003; Spergel et al. 2007). These results are consistent with an inflationary origin for the primordial perturbations, since the simplest forms of cosmic inflation produce nearly Gaussian fluctuations.

In order to understand the early Universe in more detail, however, it is necessary to measure (or at least constrain) the departures

from non-Gaussianity that inevitably arise at some level during the inflationary epoch. For example, the simplest slowly rolling single field inflation model predicts very small levels of primordial non-Gaussianity, while multifield inflation models and models with a non-standard kinetic term for the inflation may yield larger effects which could be detected in ongoing or next-generation observations (e.g. Bartolo, Matarrese & Riotto 2002; Bernardeau & Uzan 2002; Lyth, Ungarelli & Wands 2003; Alishahiha, Silverstein & Tong 2004; Arkani-Hamed et al. 2004; Bartolo et al. 2004; Dvali, Gruzinov & Zaldarriaga 2004; Battefeld & Battefeld 2007; Chen, Richard & Eugene 2007). Only when such phenomena are detected will it be possible to distinguish between the hundreds of presently viable variations on the theme of inflation by understanding the dynamical behaviour of the inflation field.

In order to model the primordial non-Gaussianity that might arise during inflation, the following simple form including quadratic corrections to the curvature perturbation Φ (Bardeen 1980) during the

*E-mail: chiaki.hikage@nottingham.ac.uk

matter era has been often adopted (Gangui et al. 1994; Verde et al. 2000; Komatsu & Spergel 2001):

$$\Phi = \phi + f_{\text{NL}}(\phi^2 - \langle \phi^2 \rangle), \quad (1)$$

where ϕ represents an auxiliary random-Gaussian field and f_{NL} characterizes the amplitude of a quadratic correction to the curvature perturbations in a dimensionless way. In principle, f_{NL} could be scale-dependent, but present observations are not sufficiently sensitive to detect any such variation, so a constant f_{NL} remains a useful parametrization of the level of non-Gaussianity. Recent analyses of the angular bispectrum for *WMAP* provide strong constraints on f_{NL} to lie in the range from -54 to 114 at the 95 per cent confidence level (Komatsu et al. 2003; Creminelli et al. 2006; Spergel et al. 2007).

The large-scale structure (LSS) of the distribution of galaxies in the Universe provides another potentially powerful probe of primordial non-Gaussianity (Fry & Scherrer 1994; Chodorowski & Bouchet 1996; Verde et al. 2000; Scoccimarro, Sefusatti & Zaldarriaga 2004; Hikage, Komatsu & Matsubara 2006; Sefusatti & Komatsu 2007). The three-dimensional spatial information arising from LSS is potentially a richer source information about primordial non-Gaussianity than the two-dimensional information arising from the CMB. For example, constraints from upcoming cluster surveys should be comparable with present CMB limits and those from galaxy surveys, which could be as tight as $|f_{\text{NL}}| \sim 10$ for the planned surveys and $|f_{\text{NL}}| \sim 0.2$ for an all-sky survey of galaxies up to redshift $z = 5$ (Sefusatti & Komatsu 2007; Dalal et al. 2007). A variety of large-scale projects of LSS observation covering Gpc³ volumes are being proposed, such as an extension of the Sloan Digital Sky Survey; APO-LSS survey; The Hobby-Eberly Dark Energy Experiment (HETDEX) (Hill et al. 2004); Wide-Field Multi-Object Spectrograph (WFOS) (Glazebrook et al. 2005); and the Cosmic Inflation Probe (CIP) mission (Melnick et al. 2004). It is consequently important to study the optimal way to extract information about primordial non-Gaussianity from such surveys.

The statistical analysis of non-Gaussianity has been mainly performed through the calculation of the bispectrum (Verde et al. 2000; Scoccimarro et al. 2004; Sefusatti & Komatsu 2007). Strong motivation for this is that the bispectrum is the simplest statistical function that can measure quadratic non-linearity (e.g. Watts & Coles 2003). Although the quadratic model provides an extremely useful benchmark for statistical analysis techniques, one must always bear in mind that there are many different ways for a random field to be non-Gaussian. In general, there is no one statistic that completely characterizes the statistical nature of a non-Gaussian random field, so a battery of higher-order statistics must be deployed. In particular, when the full nature of non-Gaussianity is virtually unknown, such as is really the case for primordial perturbations, the theoretical model assumed should be validated before its parameters are constrained. Different statistics reflect different aspects of non-Gaussianity so the use of different statistics plays a vital role in this kind of consistency check.

In this paper we use a set of invariant characteristics of the topology of the cosmic web, known as the Minkowski functionals (MFs). These have already been used to describe the morphological properties of cosmic density fields in a variety of contexts (Mecke, Buchert & Wagner 1994; Schmalzing & Buchert 1997; Schmalzing & Górski 1998; Hikage et al. 2003). Four MFs are defined in three-dimensional density fields such as LSS: the volume fraction (V_0); surface area (V_1); mean curvature (V_2); and Euler characteristic (V_3).

Using a perturbative approach, Hikage et al. (2006) derived analytical formulae for the behaviour of the MFs for LSS including primordial non-Gaussianity [as a function of f_{NL} as given in equa-

tion (1)], in addition to the non-Gaussianity due to non-linear gravity and galaxy biasing. The validity of the perturbative analysis is, however, limited to the weakly non-linear regime. Smaller-scale modes also contain rich information about the primordial density fields, and this could help place more stringent constraints on primordial non-Gaussianity. In this paper, we use high-resolution N -body simulations to study the effect of primordial non-Gaussianity on the MFs from the mildly to strongly non-linear regime. There are two reasons for using the full numerical analysis: one is to see how well the perturbative formulae describe the simulated MFs to check their applicability; and the other is to study how the primordial non-Gaussian effect behaves in the strongly non-linear regime and thus to estimate the significance of the effect on the MFs.

This paper is organized as follows. In Section 2, we review the perturbative formulae for the MFs. The details of the N -body simulations and the computing method of the MFs are summarized in Section 3. In Section 4, we compare the perturbative formulae of MFs with simulated results to study the primordial non-Gaussian effect in non-linear regime. Section 5 is devoted to the summary and conclusions.

2 PERTURBATION THEORY

We define the MFs of density fields for a given threshold $v \equiv \delta/\sigma_0$, where δ is the density fluctuation, which has zero mean, and $\sigma_0 \equiv \langle \delta^2 \rangle^{1/2}$ is its standard deviation. The k th MF $V_k(v)$ can be written separately with the amplitude A_k and the function of v , $v_k(v)$, as

$$V_k(v) = A_k v_k(v). \quad (2)$$

The amplitude part A_k , which depends only on the power spectrum $P(k, z)$ of the three-dimensional fluctuation field δ at redshift z , is given by

$$A_k = \frac{1}{(2\pi)^{(k+1)/2}} \frac{\omega_3}{\omega_{3-k}\omega_k} \left[\frac{\sigma_1(z)}{\sqrt{3}\sigma_0(z)} \right]^k, \quad (3)$$

where $\omega_k \equiv \pi^{k/2}/\Gamma(k/2 + 1)$ gives $\omega_0 = 1$, $\omega_1 = 2$, $\omega_2 = \pi$, and $\omega_3 = 4\pi/3$. The quantity σ_i^2 characterizes the variance of fluctuating fields for $i = 0$ and that of their derivatives for $i = 1$ is given by

$$\sigma_i^2(z) \equiv \int_0^\infty \frac{k^2 dk}{2\pi^2} k^{2i} P(k, z) W^2(kR), \quad (4)$$

where W represents a smoothing kernel. Throughout this paper, we adopt a Gaussian kernel $W^2 = \exp[-(kR)^2]$, where R represents the smoothing scale.

Matsubara (2003) derives the second-order perturbative formulae of the MFs using the multivariate Edgeworth expansion. According to the formulae, the function $v_k(v)$ is written with the Gaussian part $v_k^{(G)}$ and the leading part of the non-Gaussian term Δv_k as

$$v_k(v) = v_k^{(G)}(v) + \Delta v_k(v), \quad (5)$$

$$v_k^{(G)}(v) = e^{-v^2/2} H_{k-1}(v), \quad (6)$$

$$\Delta v_k(v) = e^{-v^2/2} \left[\frac{1}{6} S^{(0)} H_{k+2}(v) + \frac{k}{3} S^{(1)} H_k(v) + \frac{k(k-1)}{6} S^{(2)} H_{k-2}(v) \right] \sigma_0, \quad (7)$$

where $H_n(v)$ denote the Hermite polynomials. The leading-order non-Gaussian term $\Delta v_k(v)$ is calculated when the three ‘skewness parameters’ $S^{(i)}$ are given.

The three skewness parameters $S^{(i)} (i = 0, 1 \text{ and } 2)$ are computed by integrating the bispectrum $B(k_1, k_2, k_3, z)$ over k_1, k_2 , and $\mu \equiv (\mathbf{k}_1 \cdot \mathbf{k}_2)/(k_1 k_2)$ with appropriate weights as (Hikage et al. 2006):

$$S^{(0)}(z) = \frac{1}{8\pi^4 \sigma_0^4(z)} \int_0^\infty dk_1 \int_0^\infty dk_2 \int_{-1}^1 d\mu k_1^2 k_2^2 \times B(k_1, k_2, k_{12}, z) W(k_1 R) W(k_2 R) W(k_{12} R), \quad (8)$$

$$S^{(1)}(z) = \frac{1}{16\pi^4 \sigma_0^2(z) \sigma_1^2(z)} \int_0^\infty dk_1 \int_0^\infty dk_2 \int_{-1}^1 d\mu \times k_1^2 k_2^2 (k_1^2 + k_2^2 + \mu k_1 k_2) B(k_1, k_2, k_{12}, z) \times W(k_1 R) W(k_2 R) W(k_{12} R), \quad (9)$$

$$S^{(2)}(z) = \frac{3}{16\pi^4 \sigma_1^4(z)} \int_0^\infty dk_1 \int_0^\infty dk_2 \int_{-1}^1 d\mu \times k_1^4 k_2^4 (1 - \mu^2) B(k_1, k_2, k_{12}, z) \times W(k_1 R) W(k_2 R) W(k_{12} R), \quad (10)$$

where $k_{12} \equiv |\mathbf{k}_1 + \mathbf{k}_2| = (k_1^2 + k_2^2 + 2\mu k_1 k_2)^{1/2}$.

Throughout this paper, we neglect the non-Gaussianity arising from the non-linearity in relationship between galaxy counts and mass (i.e. galaxy biasing) so as to keep the analysis as simple as possible. The bispectrum B for the matter density fluctuation is then given by

$$B(k_1, k_2, k_3, z) = B_{\text{pri}}(k_1, k_2, k_3, z) + B_{\text{gr}}(k_1, k_2, k_3, z), \quad (11)$$

where B_{pri} and B_{gr} represent the contributions from primordial non-Gaussianity and non-linearity in gravitational clustering, respectively:

$$B_{\text{pri}}(k_1, k_2, k_3, z) \equiv \frac{2f_{\text{NL}}}{D(z)} \left[\frac{P(k_1, z) P(k_2, z) M(k_3)}{M(k_1) M(k_2)} + (\text{cyc.}) \right], \quad (12)$$

$$B_{\text{gr}}(k_1, k_2, k_3, z) \equiv 2[F_2(\mathbf{k}_1, \mathbf{k}_2) P(k_1, z) P(k_2, z) + (\text{cyc.})], \quad (13)$$

where $D(z)$ is the growth rate of linear density fluctuations normalized such that $D(z) \rightarrow 1/(1+z)$ during the matter era. The function $M(k)$ and $F_2(\mathbf{k}_1, \mathbf{k}_2)$ are time-independent kernels describing mode-coupling due to non-linear clustering of matter density fluctuations in the weakly non-linear regime. These are given by

$$M(k) \equiv \frac{2}{3} \frac{k^2 T(k)}{\Omega_m H_0^2}, \quad (14)$$

$$F_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{5}{7} + \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{2k_1 k_2} \left(\frac{k_1}{k_2} + \frac{k_2}{k_1} \right) + \frac{2}{7} \frac{(\mathbf{k}_1 \cdot \mathbf{k}_2)^2}{k_1^2 k_2^2}. \quad (15)$$

We adopt the linear transfer function $T(k)$ by Eisenstein & Hu (1999). In comparison with numerical simulations, we use the power spectrum of the simulations (the details are explained in the next section) at $z^* = 76.97$ for a theoretical input of the power spectrum $P(k, z^*)$ and then give the power spectrum at z as

$$P(k, z) = \frac{D^2(z)}{D^2(z^*)} P(k, z^*). \quad (16)$$

3 METHODOLOGY

3.1 Numerical simulations with primordial non-Gaussianity

The N -body simulations with primordial non-Gaussianity that we use for this analysis are those described in Grossi et al. (2007). These simulations employ 800^3 dark matter particles in a periodic cubic box with a side length of $0.5 h^{-1}$ Gpc. The cosmology of our simulations is a flat Λ cold dark matter model with the mass density parameter $\Omega_m = 0.3$, baryon density parameter $\Omega_b = 0.04$, Hubble parameter $h = 0.7$, primordial power-law index $n_s = 1$, and $\sigma_8 = 0.9$.

The initial particles are perturbed from an initially homogeneous ‘glass-like’ distribution. The primordial non-Gaussianity is incorporated into a Gaussian-random field with the above cosmology in the form of equation (1). Grossi et al. (2007) explored seven different scenarios with $f_{\text{NL}} = 0, \pm 100, \pm 500$ and ± 1000 . We have analysed all of these simulations, but for brevity in this paper we only present results for the Gaussian simulation with $f_{\text{NL}} = 0$ and the two extreme non-Gaussian cases $f_{\text{NL}} = \pm 1000$; results for the other simulations with $f_{\text{NL}} = \pm 100$ are intermediate, as expected.

After Fourier-transforming the primordial non-Gaussian field, the dark matter particles are displaced on the initial grid assuming the Zel’dovich approximation. The simulations are started at $z \approx 100$ and the subsequent gravitational evolution is simulated with the GADGET-2 code (Springel 2005). The Triangular-Shaped Cloud method is used to assign densities on to 512^3 grids. After Fourier-transforming the grid data, we multiply by the Gaussian kernel $\exp[-(kR)^2]$, and then transform them back to real space.

It is instructive first to examine the visual morphology of the clustering pattern. Fig. 1 shows maps of slices of the mass density field with $f_{\text{NL}} = 0$ (middle-row panels) and the relative residuals between $f_{\text{NL}} = \pm 1000$ and 0 (left-hand and right-hand panels). The residual for the map with $f_{\text{NL}} = x$, $\Delta\rho_x$, is calculated at each pixel as

$$\Delta\rho_x = \frac{(\rho_x - \rho_0)}{\rho_0}, \quad (17)$$

where ρ_x is the number density of mass particles for the map with $f_{\text{NL}} = x$. The field is smoothed with a Gaussian filter 10 pixels wide (i.e. $9.8 h^{-1}$ Mpc). The redshifts of the maps are 5.15, 2.13 and 0 from the top to bottom panel, respectively. Similar density structures in the mass distribution appear in the residual maps with their contrast at same (inverse) sign for positive (negative) values of f_{NL} . For example, a large void structure at the right-hand centre in the density map also appears in the residual maps. This is because the higher density region is initially more (less) enhanced in the positive (negative) f_{NL} , as predicted by the local model of primordial non-Gaussianity in equation (1).

3.2 Computation of MFs

The computational method we use for calculating MFs of data defined on a grid is based on ideas from integral geometry, rather than the alternative more cumbersome approach of using the differential properties of bounding surfaces. In our case, the calculation reduces to counting the numbers of vertices, edges and sides of the elementary cells covering the structure (Coles, Davies & Pearson 1996; Schmalzing & Buchert 1997). The range of v is from -3.6 to 3.6 with an equal binning width of 0.2. The MFs measured from numerical simulations often deviate from analytical predictions even for Gaussian realizations due to subtle pixelization effects. However, as pointed out by Hikage et al. (2006), pixelization effects become

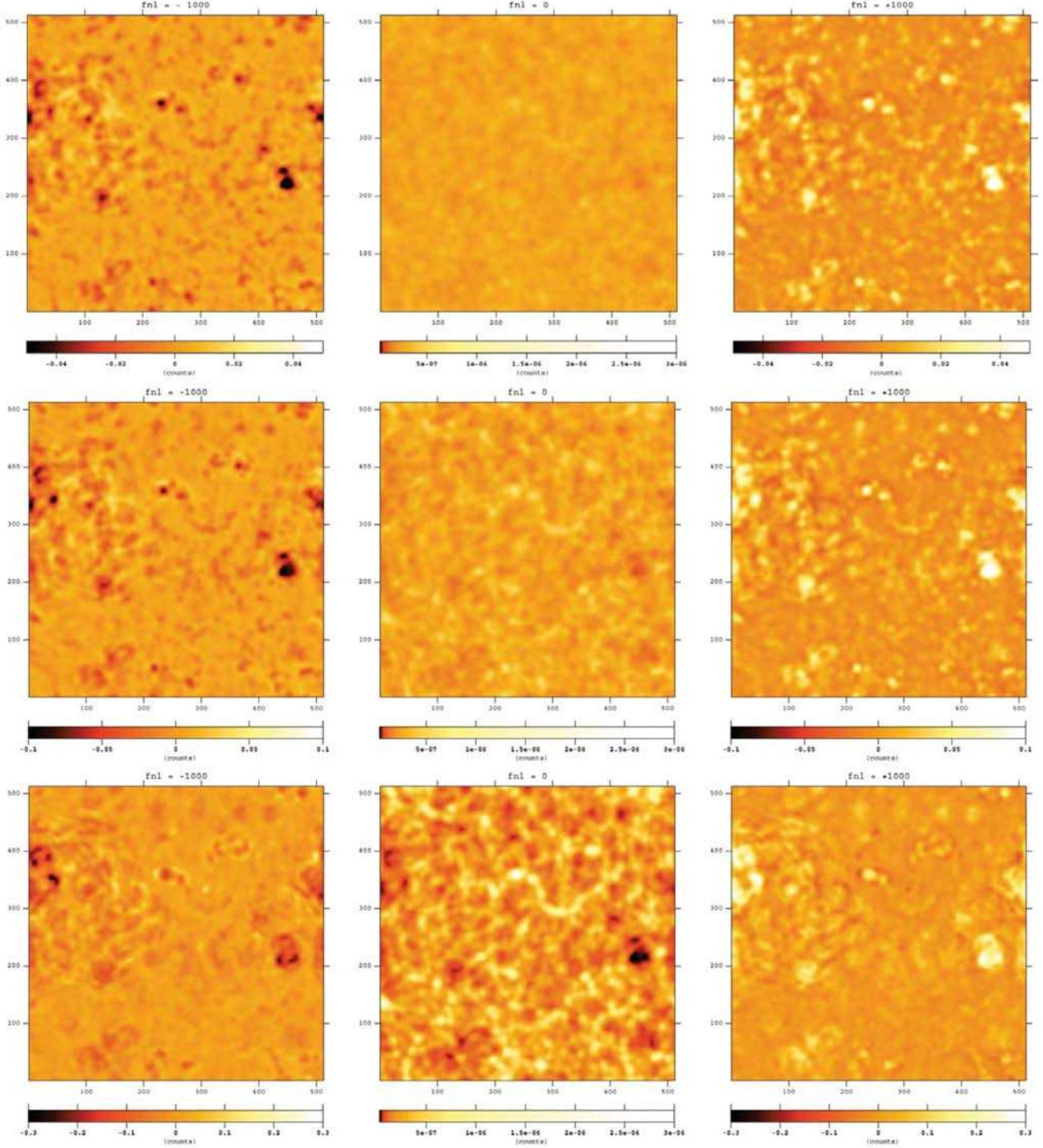


Figure 1. Slice maps of simulated mass density fields at $z = 5.15$ (top panel), $z = 2.13$ (middle panel) and $z = 0$ (bottom panel). The number of pixels at a side length is 512 ($500 h^{-1}$ Mpc) and that of the thickness is 32 ($31.25 h^{-1}$ Mpc). The panels in the middle row show the log of the projected density smoothed with a Gaussian filter of 10 pixel width, corresponding to $9.8 h^{-1}$ Mpc. The left-hand and right-hand panels are the relative residuals for the $f_{\text{NL}} = \pm 1000$ runs (equation 17). Each panel has the corresponding colour bar and the ranges considered are different from panel to panel.

negligible when computing the difference between Gaussian and non-Gaussian MFs. Therefore, we focus on $\Delta v_k(v_i)$ (i denoting the binning number of v) that we compute as follows.

(i) We compute the MFs for non-Gaussian simulation data V_k and then divide them by their amplitudes A_k (equation [3]) to obtain

normalized MFs V_k . σ_0 and σ_1 in A_k are computed from the density fields of the simulations.

(ii) The MFs for Gaussian fields are computed in the same way and then divided by their amplitudes A_k where the values of σ_0 and σ_1 are computed from each realization. The same cosmological parameters as the N -body simulations are adopted. The normalized

MFs $v_k^{(G)}$ are estimated by averaging MFs over 10 Gaussian realizations.

(iii) The difference ratio Δv_k is computed by

$$\Delta v_k = v_k - v_k^{(G)}. \quad (18)$$

4 RESULTS

In this section, we explore two different but related issues. The first is whether the non-linear behaviour seen in numerical simulations matches the predictions of analytical approaches. The second is whether it is possible to separate the effects of non-linear evolution from primordial non-Gaussianity to a sufficient extent for this method to be useful in practice.

4.1 Agreement with perturbative formulae in the weakly non-linear regime

Fig. 2 shows examples of MFs V_k (left-hand panels) and the difference ratio Δv_k (right-hand panels) for simulated mass distributions in the weakly non-linear regime. We smooth on a scale $R = 10h^{-1}\text{Mpc}$ which, at $z = 3.96$, marks the transition to the non-linear regime since the variance of the smoothed density fluctuation $\sigma_0 \simeq 0.1$. The different symbols show the different f_{NL} of 0 and ± 1000 . The error bars represent the sample variance estimated from 1000 Gaussian realizations with the same R, z and box-size as the simulations. The perturbative formulae discussed above are plotted with the lines for comparison. Results for the simulations with $f_{\text{NL}} = \pm 100$ and ± 500 are found to be linearly scaled between those with $f_{\text{NL}} = 0$ and ± 1000 .

The theoretical curves reproduce the features of the simulated MFs very well. We quantitatively estimate the agreement between the simulation results $\Delta v_k^{(\text{SIM})}(v_i)$ and the perturbative formulae $\Delta v_k^{(\text{PT})}(v_i)$ by calculating the root-mean-square (rms) differences averaged over i . Table 1 lists the differences for each MF at different redshifts z (but R is fixed to be $10h^{-1}\text{Mpc}$). The differences are less than a few per cent relative to the amplitude of each MF (equation 3) when $\sigma_0 < 0.1$ and remains at the 10 per cent level when $\sigma_0 \sim 0.2$. We also estimate the rms differences divided by the rms of $\Delta v_k^{(\text{SIM})}(v_i)$ averaged over i . These quantities represent the extent to which the theoretical predictions improve going from linear theory to (second-order) perturbation theory. The differences between the second-order perturbative predictions and the numerical simulations are 0.15 \sim 0.41 times smaller than those corresponding to linear theory at $\sigma_0 < 0.1$. These results are consistent with the previous analysis by Nakagami et al. (2004).

The differences between theory and simulations are quite small compared to the sample variance. However, there is a systematic feature, seen in the asymmetry of V_0 and V_2 with respect to $v = 0$; the perturbative predictions are symmetric. There are three possible explanations for this effect. One is that higher-order contributions – that is, beyond second order – are significant. Another possibility arises from the use of the Zel’dovich approximation to set the initial conditions of the simulations, which may be responsible for an extra contribution to higher-order statistical properties of clustering arising from transients (Crocce, Sebastián & Scoccimarro 2006). The other reason is the fact that the multivariate Edgeworth expansion which is the basis of perturbation formulae has a limited range of validity, especially at values of v larger than unity (Bernardeau & Kofman 1995). These effects must be considered carefully when comparing with real survey results.

4.2 Non-linear evolution and primordial non-Gaussianity

In Fig. 3, we focus on the differences between Δv_k with $f_{\text{NL}} = 1000$ and that with $f_{\text{NL}} = 0$ at $z = 0$. The perturbative predictions are also plotted for comparison. The deviation from the perturbative predictions becomes significant as the smoothing scale R is smaller (σ_0 increases) due to the primordial non-Gaussian effect coupled with non-linear gravity. The increase in deviations at larger σ_0 is also seen quantitatively in Table 1. The shape of the deviation is skewed to the positive side of v with a higher peak at $v = -1/\sigma_0$ (the number density is zero), while the overall amplitude of the deviation Δv_k is roughly the same as that from the perturbative predictions.

It is interesting to estimate the sensitivity of the MFs to primordial non-Gaussianity in the non-linear regime, because the effect of primordial non-Gaussianity on the MFs should become increasingly significant as the sample variance decreases, that is, at smaller smoothing scales. The MFs are, however, strongly correlated with each other among different bins of the threshold v and it is therefore necessary to take into account their covariance when estimating the significance of the primordial non-Gaussian effect with, for example, χ^2 statistics. If the covariances among different bins were not considered, one would overestimate the value of χ^2 as the total number of bins increases. When the field follows nearly Gaussian statistics, the covariance matrix is well approximated with the one numerically estimated from a large number of Gaussian realizations. (Komatsu et al. 2003; Hikage et al. 2006). When the field is non-linearly evolved, it is an exceptionally time-consuming process to generate enough number of realizations to compute the inverse matrix of the covariance (the number of realizations must be larger than the degree-of-freedom at least).

Instead of calculating the covariance matrix directly, therefore, we instead estimate the amount of information contained in each MF as a function of v . For this purpose, we calculate the effective number of bins, N_{eff} , for each MF and for all MFs combined as follows:

$$N_{\text{eff}} = N_{\text{bin}} \frac{\sum_{i,j}^{N_{\text{bin}}} \Delta v_i^{(\text{PT})} (C^{-1})_{ij} \Delta v_j^{(\text{PT})}}{\sum_i^{N_{\text{bin}}} \Delta v_i^{(\text{PT})} C_{ii}^{-1} \Delta v_i^{(\text{PT})}}, \quad (19)$$

where i and j denote the binning number of different v and different kinds of MFs and N_{bin} denotes the total number of bins. The covariance matrix $C_{ij} = \langle \Delta v_i \Delta v_j \rangle$ is computed from 1000 Gaussian realizations with the same cosmological parameters and the same box-size as those of the N -body simulations. As N_{bin} is increased in a fixed range of v from -3.6 to 3.6, the values of N_{eff} converges to 2, 6, 8 and 12 for each MF from $k = 0$ to 3 and then 12 for all MFs combined. The results indicate that the correlations among different bins of v is very strong for V_0 and that higher k th MFs have more independent information as a function of v .

Applying the value of N_{eff} for non-linearly evolved simulations, we calculate the χ^2 values of the primordial non-Gaussian effect on MFs as a function of f_{NL} as

$$\chi^2(f_{\text{NL}}) = \frac{N_{\text{eff}}}{N_{\text{bin}}} \sum_i^{N_{\text{bin}}} \frac{(\Delta v_i^{(\text{SIM})}(f_{\text{NL}}) - \Delta v_i^{(\text{SIM})}(f_{\text{NL}} = 0))^2}{\langle \Delta v_i^{(\text{SIM})}(f_{\text{NL}} = 0)^2 \rangle}. \quad (20)$$

The variance $\langle \Delta v_i^{(\text{SIM})}(f_{\text{NL}} = 0)^2 \rangle$ is estimated from 10 realizations of N -body simulations with Gaussian initial conditions (the cosmological parameters and simulation box-size are the same as for the N -body simulations). The normalized MFs $\Delta v_k(f_{\text{NL}})$ at arbitrary f_{NL} are linearly interpolated using the simulation results with $f_{\text{NL}} = 0$ and 1000. We confirm that the linear interpolation works well using simulations with $|f_{\text{NL}}| = 100$ and 500.

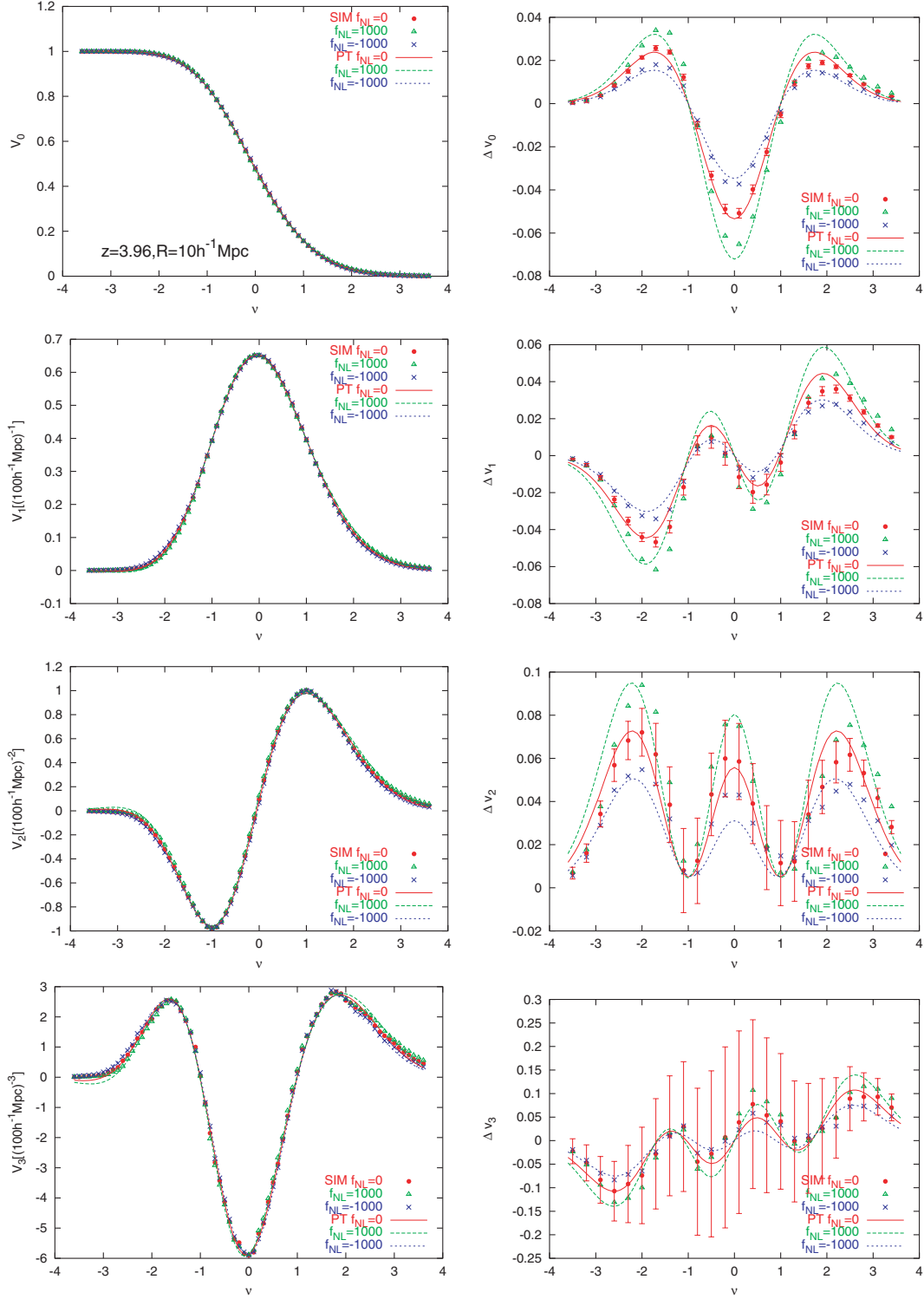


Figure 2. Four MFs V_k (left-hand panel) and their difference ratios Δv_k (right-hand panel) for the simulated mass density fields at $z = 3.96$, with $f_{\text{NL}} = 0$ (filled circles), 1000 (open triangles) and -1000 (crosses). The simulated fields are smoothed with a Gaussian window function at the scale $R = 10 h^{-1} \text{Mpc}$. The error bars denote the sample variance estimated from 1000 Gaussian realizations with the same z , R and box-size as the simulations. For comparison, the theoretical expectations from perturbation theory (equations 2 and 7) are plotted with the lines.

Table 2 lists the value of f_{NL} at different R when the effect of the primordial non-Gaussianity is comparable to the sample variance, that is, $\chi^2 = 1$. The volume of the simulation box-size is $0.125(h^{-1} \text{Gpc})^3$, which is less than half the volume of the SDSS

main galaxy sample $0.3(h^{-1} \text{Gpc})^3$. As the smoothing scale decreases, the primordial non-Gaussianity becomes significant. At $R = 5 h^{-1} \text{Mpc}$, the primordial non-Gaussianity with $f_{\text{NL}} = 50$ is comparable to the sample variance and then corresponds to the

Table 1. Root-mean-square differences between simulated MFs and perturbative formulae at different z and f_{NL} with the corresponding σ_0 . The smoothing scale is fixed at $R = 10 h^{-1}$ Mpc.

z	f_{NL}	σ_0	$\sqrt{\langle(\Delta v_k^{(\text{SIM})} - \Delta v_k^{(\text{PT})})^2\rangle}$				$\sqrt{\langle(\Delta v_k^{(\text{SIM})} - \Delta v_k^{(\text{PT})})^2\rangle / \langle(\Delta v_k^{(\text{SIM})})^2\rangle}$			
			V_0	V_1	V_2	V_3	V_0	V_1	V_2	V_3
5.15	0	0.080	0.003	0.006	0.009	0.024	0.15	0.27	0.24	0.41
5.15	1000	0.080	0.005	0.008	0.013	0.029	0.17	0.27	0.24	0.38
5.15	-1000	0.080	0.003	0.006	0.012	0.024	0.33	0.44	0.40	0.54
3.96	0	0.099	0.003	0.006	0.009	0.023	0.14	0.24	0.20	0.37
3.96	1000	0.099	0.006	0.010	0.014	0.031	0.20	0.34	0.26	0.41
3.96	-1000	0.099	0.003	0.004	0.008	0.020	0.14	0.20	0.23	0.41
2.13	0	0.16	0.007	0.014	0.021	0.044	0.18	0.34	0.29	0.44
2.13	1000	0.16	0.011	0.021	0.029	0.057	0.23	0.40	0.34	0.48
2.13	-1000	0.16	0.004	0.010	0.017	0.034	0.15	0.31	0.27	0.40
0.96	0	0.24	0.015	0.030	0.043	0.081	0.26	0.48	0.41	0.58
0.96	1000	0.24	0.020	0.039	0.055	0.10	0.31	0.54	0.48	0.65
0.96	-1000	0.24	0.010	0.023	0.034	0.065	0.22	0.43	0.36	0.51
0	0	0.38	0.035	0.067	0.095	0.17	0.40	0.68	0.63	0.86
0	1000	0.38	0.042	0.078	0.11	0.19	0.44	0.73	0.69	0.93
0	-1000	0.38	0.028	0.058	0.083	0.15	0.36	0.64	0.58	0.80

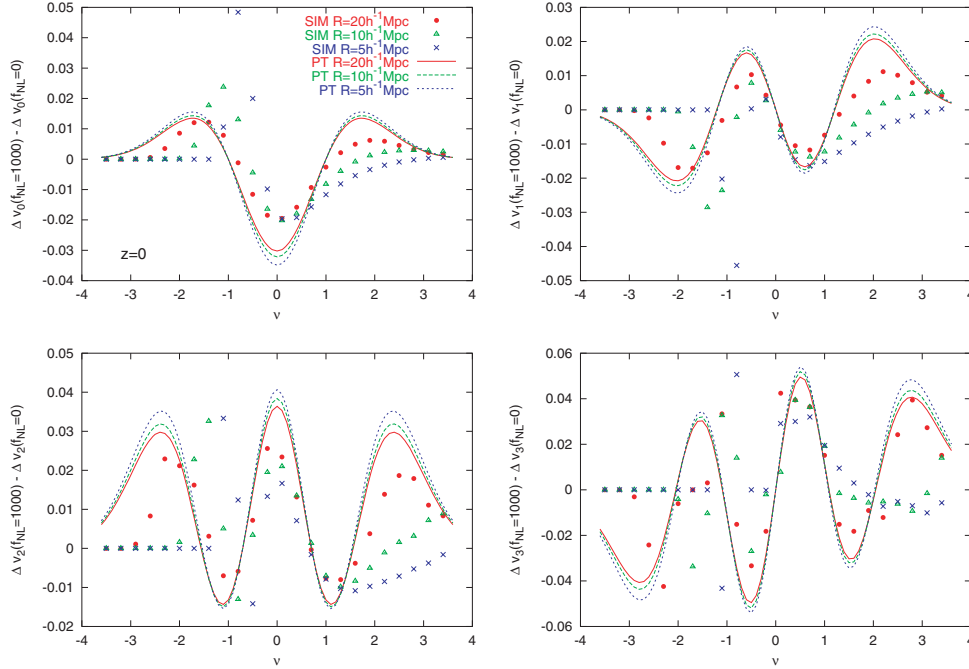


Figure 3. The difference of Δv_k with $f_{\text{NL}} = 1000$ from those obtained with Gaussian initial conditions $\Delta v_k(f_{\text{NL}} = 0)$ at $z = 0$ for different smoothing scales $R = 20 h^{-1}$ Mpc ($\sigma_0 = 0.17$), $10 h^{-1}$ Mpc ($\sigma_0 = 0.38$), and $5 h^{-1}$ Mpc ($\sigma_0 = 0.74$). Simulated results averaged over three bins are plotted with the symbols and the perturbative formulae are also plotted with the lines.

present observational constraints from *WMAP*. Note that the detectability of primordial non-Gaussianity from actual observations is, however, strongly dependent on the uncertainty of the cosmological parameters and the galaxy biasing, which we have not attempted to model in detail.

5 SUMMARY AND CONCLUSIONS

We have studied the imprint of primordial non-Gaussianity on the topological properties of LSS using the MFs. Characterizing pri-

mordial non-Gaussianity as a quadratic correction to the primordial potential fluctuation with constant amplitude f_{NL} , we compare the MFs with different values of f_{NL} from the mildly to the strongly non-linear regime using high-resolution N -body simulations. Perturbative formulae of the MFs based on the multivariate Edgeworth expansion well reproduce the MFs of simulated mass density fields in the weakly non-linear regime. When the amplitude of the density fluctuation $\sigma_0 < 0.1$ and $|f_{\text{NL}}| < 1000$, the deviations of the perturbative formulae from simulations are less than a few per cent of the amplitude of each MF. They are also 10 ~ 40 per cent with respect to the non-Gaussian contributions alone.

Table 2. The values of f_{NL} at $\chi^2 = 1$ when the effect of the primordial non-Gaussianity is comparable to the sample variance of mass density field for a different smoothing scale R (equation 20). The values of N_{eff} for each MF V_k are 2, 6, 8 and 12 from $k = 0$ to 3 and 12 for all MFs combined. The volume of the density field is a cube at a length $0.5 h^{-1}$ Gpc and the redshift is 0. The other cosmological parameters are fixed to be fiducial values. The effective number of bins, N_{eff} (equation 19), is also listed in the last line.

R (h^{-1} Mpc)	f_{NL} at $\chi^2 = 1$				
	V_0	V_1	V_2	V_3	All MFs
30	770	480	520	370	350
20	420	300	310	210	210
10	190	180	140	150	110
5	90	80	90	60	50

As the fluctuations become more strongly non-linear, the simulated MFs begin to deviate significantly from the perturbative predictions owing to non-linear gravitational evolution. In order to include small-scale information in realistic cosmological data sets, detailed numerical analysis is therefore essential.

When we include information from smaller scale fluctuations, the effects of primordial non-Gaussianity are indeed significant. Using χ^2 statistics, we find that the primordial non-Gaussianity with $f_{\text{NL}} = 50$ has a significance level corresponding to 1σ , considering the sample variance of mass density fields at $R = 5 h^{-1}$ Mpc with a volume of $0.125(h^{-1} \text{ Gpc})^3$. This implies that measuring the MFs in a SDSS-like survey could constrain f_{NL} at a level comparable with present CMB limits. This is an interesting result, since other observations, like the cluster abundance, that can effectively constrain f_{NL} at high redshifts become useless at $z = 0$ when non-Gaussian features generated by non-linear dynamics completely obliterate primordial ones (Grossi et al. 2007; Kang, Norberg & Silk 2007).

The actual detectability of the primordial non-Gaussianity is, however, strongly dependent on the degeneracy between the cosmological parameters and the primordial non-Gaussian effect. Understanding the properties of the galaxy biasing is also very important in determining the primordial non-Gaussianity accurately. We will consider this issue in a forthcoming paper.

ACKNOWLEDGMENTS

We thank the anonymous referee for helpful comments. We thank Takahiko Matsubara for useful discussions. CH acknowledges support from the Particle Physics and Astronomy Research Council grant number PP/C501692/1. Computations have been performed on the IBM-SP5 at Consorzio Interuniversitario del Nord-Est per il Calcolo Automatico (CINECA), Bologna, with CPU time assigned under an INAF-CINECA grant and on the IBM-SP4 machine at the ‘Rechenzentrum der Max-Planck-Gesellschaft’ at the Max-Planck-Institut für Plasmaphysik with CPU time assigned to the MPA. We acknowledge financial contribu-

tion from contracts ASI-INAF I/023/05/0, ASI-INAF I/088/06/0 and INFN PD51.

REFERENCES

- Alishahiha M., Silverstein E., Tong D., 2004, *Phys. Rev. D*, 70, 123505
 Arkani-Hamed N., Creminelli P., Mukohyama S., Zaldarriaga M., 2004, *J. Cosmology Astroparticle Phys.*, 4, 1
 Bardeen J. M., 1980, *Phys. Rev. D*, 22, 1882
 Bartolo N., Matarrese S., Riotto A., 2002, *Phys. Rev. D*, 65, 103505
 Bartolo N., Komatsu E., Matarrese S., Riotto A., 2004, *Phys. Rept.*, 402, 103
 Battefeld D., Battefeld T., 2007, *J. Cosmology Astroparticle Phys.*, 5, 12
 Bernardeau F., Kofman L., 1995, *ApJ*, 443, 479
 Bernardeau F., Uzan J.-P., 2002, *Phys. Rev. D*, 66, 103506
 Chen X., Richard E., Eugene A. L., 2007, *J. Cosmology Astroparticle Phys.*, 6, 23
 Chodorowski M. J., Bouchet F. R., 1996, *MNRAS*, 279, 557
 Coles P., Davies A. G., Pearson R. C., 1996, *MNRAS*, 281, 1375
 Creminelli P., Nicolis A., Senatore L., Tegmark M., Zaldarriaga M., 2006, *J. Cosmology Astroparticle Phys.*, 05, 004
 Crocce M., Sebastián P., Scoccimarro R., 2006, *MNRAS*, 373, 369
 Dalal N., Doré O., Huterer D., Shirokov A., 2007, preprint (arXiv:0710.4560)
 Dvali G., Gruzinov A., Zaldarriaga M., 2004, *Phys. Rev. D*, 69, 083505
 Eisenstein D. J., Hu W., 1999, *ApJ*, 511, 5
 Fry J. N., Scherrer R. J., 1994, *ApJ*, 429, 36
 Gangui A., Lucchin F., Matarrese S., Mollerach S., 1994, *ApJ*, 430, 447
 Glazebrook K., Eisenstein D., Dey A., Nichol R., The WFMOS Feasibility Study Dark Energy Team, 2005, preprint (astro-ph/0507457)
 Grossi M., Dolag K., Branchini E., Matarrese S., Moscardini L., 2007, *MNRAS*, 382, 1261
 Hikage C. et al., 2003, *PASJ*, 55, 911
 Hikage C., Komatsu E., Matsubara T., 2006, *ApJ*, 653, 11
 Hill G. J., Gebhardt K., Komatsu E., MacQueen P. J., 2004, in Allen R. E., Nanopoulos D. V., Pope C. N., eds, *AIP Conf. Proc. Vol. 743, The New Cosmology: Conference on Strings and Cosmology*. Am. Inst. Phys., New York, p. 224
 Kang X., Norberg P., Silk J., 2007, *MNRAS*, 376, 343
 Komatsu E., Spergel D. N., 2001, *Phys. Rev. D*, 63, 63002
 Komatsu E. et al., 2003, *ApJS*, 148, 119
 Lyth D. H., Ungarelli C., Wands D., 2003, *Phys. Rev. D*, 67, 23503
 Matsubara T., 2003, *ApJ*, 584, 1
 Mecke K. R., Buchert T., Wagner H., 1994, *A&A*, 288, 697
 Melnick G. J., Fazio G. G., Tolls V., Jaffe D. T., Gebhardt K., Bromm V., Komatsu E., Woodruff R. A., 2004, *BAAS*, 36, 1509
 Nakagami T., Matsubara T., Schmalzing J., Jing Y., 2004, preprint (astro-ph/0408248)
 Schmalzing J., Buchert T., 1997, *ApJ*, 482, L1
 Schmalzing J., Górski K. M., 1998, *MNRAS*, 297, 355
 Scoccimarro R., Sefusatti E., Zaldarriaga M., 2004, *Phys. Rev. D*, 69, 103513
 Sefusatti E., Komatsu E., 2007, *Phys. Rev. D*, 76, 083004
 Spergel D. N. et al., 2007, *ApJS*, 170, 377
 Springel V., 2005, *MNRAS*, 364, 1105
 Verde L., Wand L., Heavens A. F., Kamionkowski M., 2000, *MNRAS*, 313, 141
 Watts P. I. R., Coles P., 2003, *MNRAS*, 338, 806

This paper has been typeset from a \LaTeX file prepared by the author.